Code Expansion Stress for Nonlinearly Supported System

The calculation of the ASME code expansion stresses in linear piping systems can be carried out by calculating the stresses during the operating load conditions and the stresses due to sustained load conditions and subtracting the second from the first. Similarly, we can apply thermal loads only and calculate the resulting stresses. However, the piping systems with one-directional resting supports are considered nonlinear. Here we cannot apply the principle of superposition in the calculation of the expansion stresses.

The ASME code expansion stress is defined as the displacement stress produced by thermal expansion. In more precise terms, it is the stress produced in the operating condition due to thermal expansion. This stress, in case of one-directional resting support, can be produced partially by the weight force in spite of the fact that it is introduced in the system due to thermal expansion.

For systems with one-directional resting supports that are active in both loading conditions (operating and sustained), we can simply consider the support as linear double acting and apply thermal load only to calculate the expansion stress.

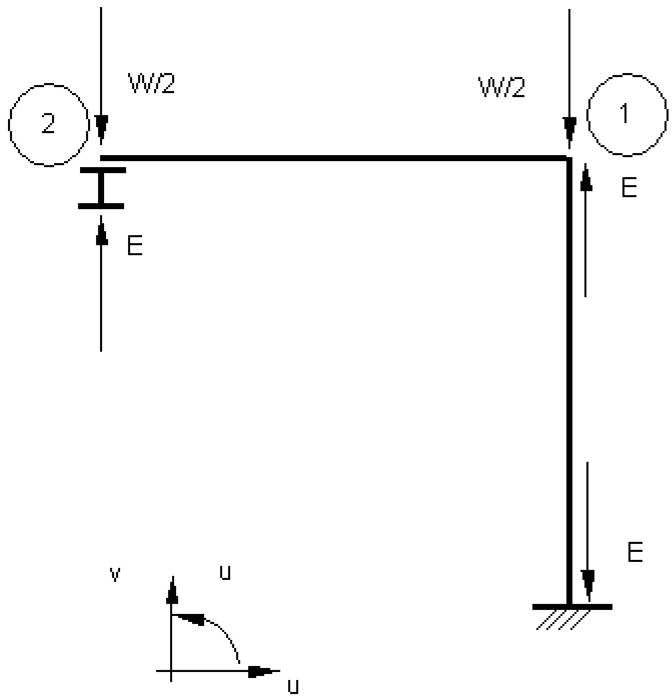
For the case of systems with one-directional supports that are active in the sustained load condition and not active in the operating load condition, a new approach has to be considered. If we eliminate the nonlinearly acting supports the system becomes linear. The nonlinear supports are then considered as part of nonlinear boundary conditions. These are boundary conditions that are dependent on the reaction of the supports in the different loading conditions. (also for the case of a supports that lifts off in thermal only case, there is a need for further consideration).

There are two approaches to deal with the problem:

- In the first approach, we consider the displacement due to the operating load condition as D1 and the displacement due to the sustained load condition as D2. We calculate the displacement D3 as: D3 = D1 – D2. We apply the displacement D3 to the system and calculate the resulting forces and stresses, which are the expansion stresses.
- In the second approach, we consider the case of sustained load condition and calculate the reaction forces of the nonlinear supports (F) that are not active in the operating load condition. Then, we apply the thermal loads together with the reaction forces reversed in sign (-F). The stresses calculated in this case shall be the expansion stresses.

These 2 methods can also be applied to the case of one-directional support, which is active in the operating and the sustained conditions. In the first, the applied displacement at the location of the support shall be zero. In the second case, we eliminate the support and apply the thermal load plus the reaction force of the support in the operating condition minus the reaction force in the sustained condition.

Consider the example shown which consists of a vertical pipe run anchored at the lower end and connected at the upper end to a horizontal pipe run. The end of this horizontal pipe run is resting on a y-stop support. Consider the case where the thermal expansion in the vertical run lifts the pipe off the resting support at the operating load condition.



The displacement vector U of the system of two nodes (1 & 2) is given by:

$$U = \{\theta_1 \ \ u_1 \ \ v_1 \ \ \theta_2 \ \ u_2 \ \ v_2\}^T \quad \text{or} \quad \{U_1 \ \ U_2 \ \ U_3 \ \ U_4 \ \ U_5 \ \ U_6\}^T$$

For the sustained condition, the applied force vector F_S is given by:

$$F_S = \{0 \quad 0 \quad -W/2 \quad 0 \quad 0 \quad (R\text{-}W/2)\}^T$$

The displacement at the resting support $U_6 = 0$, where W is the total weight of the horizontal run, R is the reaction force of the resting support and U_6 is the vertical translation at the resting support.

The stiffness matrix of the system K can be written as:

$$K = \begin{bmatrix} Kaa & Kab \\ Kba & Kbb \end{bmatrix}$$

Kaa (5x5), Kbb (1x1), Kab (5x1), Kba (1x5)

The value of the reaction force at the sustained load condition is given by:

$$R = W/2 + Kba \; \{ U_{S1}\, U_{S2}\, U_{S3}\, U_{S4}\, U_{S5} \}^T = W/2 + Kba \; U_{SA}$$

Where U_S is the displacement in the sustained condition and U_{SA} is the vector of the first 5 components of U_S. The displacement U_{SA} is obtained from:

$$Kaa \, U_{SA} = \{ 0 \quad 0 \quad -W/2 \quad 0 \quad 0 \}^T$$

For the operating load condition, the force vector F_P is given by:

$$F_P = \{ 0 \quad 0 \quad (E-W/2) \quad 0 \quad 0 \quad -W/2\}^T$$

Where E is the expansion force in the vertical run.
The displacement vector U_P of the operating load condition is given by:

$$U_P = \{ U_{P1}\, U_{P2}\, U_{P3}\, U_{P4}\, U_{P5}\, U_{P6} \}^T = \{ U_{PA} \;\; U_{P6} \}^T$$

Where U_{PA} is the vector of the first 5 components of U_P .

$$Kaa \, U_{PA} + Kab \, U_{P6} = \{ 0 \quad 0 \quad (E-W/2) \quad 0 \quad 0 \}^T$$

$$Kba \, U_{PA} + Kbb \, U_{P6} = -W/2$$

Applying the displacement U:

$$U = U_P - U_S$$

We get the following:

$$U = \{ (U_{PA} - U_{SA}) \qquad U_{P6} \}^T$$

$(U_{PA} - U_{SA})$ is a vector with dimension 5.

The stiffness matrix of the system multiplied by the displacement vector U gives the equivalent applied force F_E.

$$F_E = \begin{bmatrix} Kaa & Kab \\ Kba & Kbb \end{bmatrix} \{ U \}^T$$

$$= \{ (Kaa\,(U_{PA} - U_{SA}) + Kab\,U_{P6}) \quad (Kba(U_{PA} - U_{SA}) + Kbb\,U_{P6}) \}^T$$

The first part is a vector of dimension 5 and the second part is scalar component. Evaluating F_E we get:

$$F_E = \{ (0\text{-}0) \quad (0\text{-}0) \quad (E\text{-}W/2 + W/2) \quad (0\text{-}0) \quad (0\text{-}0) \quad (-W/2\text{-}R+W/2) \}^T$$

$$F_E = \{ 0 \quad 0 \quad E \quad 0 \quad 0 \quad -R \}^T$$

Hence, we can conclude that, applying the difference between the operating and the sustained displacement as imposed displacement to the system is equivalent to applying thermal forces and the negative of the reaction forces of the resting support in the sustained load condition. The 2 approaches produce the same results for the expansion stresses. The expansion stress calculated by both methods is equal to the stress applied in the operating load condition minus the stress applied in the sustained load condition.

Program Caesar II uses the first approach of applying the difference between the displacement in the operating condition and the sustained condition as imposed displacement to the system to calculate the expansion stress. The stress in this case is the actual stress that applies to the system due to the thermal expansion that takes place between the operating condition and the cold or sustained condition. For Triflex Program, if the pipe lifts off the support in the operating load condition, the program disregards the support in all code stress calculations. This is based on the ground that the sustained load condition is actually considered by the code to be acting during the operating condition.